Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Background
Simultaneous Linear Equations
COMPLETE SOLUTION SET

1. Given $[A] = \begin{bmatrix} 6 & 2 & 3 & 9 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 6 \end{bmatrix}$ then $[A]$ is a (an) ______________ matrix.

(A) diagonal
(B) identity
(C) lower triangular
(D) upper triangular

Solution
The correct answer is (D).

A square matrix $[A]$ is an upper triangular matrix if $a_{ij} = 0$ when $i > j$, that is, all the elements below the diagonal are zero. Note that the statement $a_{ij} = 0$ when $i > j$ implies that the matrix elements are zero for all elements where the row number is strictly greater than the column number.

2. A square matrix $[A]$ is lower triangular if

(A) $a_{ij} = 0, j > i$

(B) $a_{ij} = 0, i > j$

(C) $a_{ij} \neq 0, i > j$

(D) $a_{ij} \neq 0, j > i$

Solution

The correct answer is (A).

A $n \times n$ matrix $[A]$ is lower triangular if $a_{ij} = 0$ for $j > i$. That is, all elements above the diagonal are zero. Note that all the elements of $[A]$ for which the column number is greater than the row numbers are zero. An example of a lower triangular matrix is

$$[A] = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 6 & 3 & 0 & 0 \\ 4 & -5 & 0 & 0 \\ 2 & 9 & 3 & -3.2 \end{bmatrix}$$

3. Given

$$[A] = \begin{bmatrix} 12.3 & -12.3 & 20.3 \\ 11.3 & -10.3 & -11.3 \\ 10.3 & -11.3 & -12.3 \end{bmatrix}, \quad [B] = \begin{bmatrix} 2 & 4 \\ -5 & 6 \\ 11 & -20 \end{bmatrix}$$

then if
$[C] = [A][B]$, then
$c_{31} =$ ______________________

(A) -58.2
(B) -37.6
(C) 219.4
(D) 259.4

Solution
The correct answer is (A).

The i^{th} row and j^{th} column of the $[C]$ matrix in $[C] = [A][B]$ is calculated by multiplying the i^{th} row of $[A]$ by the j^{th} column of $[B]$, that is,

$$c_{ij} = \begin{bmatrix} a_{i1} & a_{i2} \ldots\ldots a_{ip} \end{bmatrix} \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ \vdots \\ b_{pj} \end{bmatrix}$$

$$= a_{i1}\, b_{1j} + a_{i2}\, b_{2j} + \ldots\ldots.. + a_{ip}\, b_{pj}$$

$$= \sum_{k=1}^{p} a_{ik} b_{kj}$$

Therefore,

$$c_{31} = \begin{bmatrix} 10.3 & -11.3 & -12.3 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 11 \end{bmatrix}$$

$$= (10.3 \times 2) + (-11.3 \times -5) + (-12.3 \times 11)$$

$$= 20.6 + 56.5 + -135.3$$

$$= -58.2$$

4. The following system of equations has ____________ solution(s).

$$x + y = 2$$
$$6x + 6y = 12$$

(A) infinite
(B) no
(C) two
(D) unique

Solution
The correct answer is (A).

The system of equations

$$x + y = 2 \quad (1)$$
$$6x + 6y = 12 \quad (2)$$

has an infinite number of solutions because the two equations are the same. Equation (2) is a multiple of 6 of Equation (1).

5. Consider there are only two computer companies in a country. The companies are named *Dude* and *Imac*. Each year, *Dude* keeps 1/5th of its customers, while the rest switch to *Imac*. Each year, *Imac* keeps 1/3rd of its customers, while the rest switch to *Dude*. If in 2003, *Dude* had 1/6th of the market and *Imac* had 5/6th of the market, what will be the share of *Dude* computers when the market becomes stable?

(A) 37/90
(B) 5/11
(C) 6/11
(D) 53/90

Solution

The correct answer is (B).

If D is the current market if *Dude* computers and M is the current market of *Imac* computers, and if D_n is the next year's *Dude* market and M_n is the next years *Imac* market, then since we want when the market is stable, the market share should not change from year to year.

$$\begin{bmatrix} D_n \\ M_n \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{3} \\ \frac{4}{5} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} D_n \\ M_n \end{bmatrix}$$

$$D_n = \frac{1}{5} \times D_n + \frac{2}{3} \times M_n$$

$$M_n = \frac{4}{5} D_n + \frac{1}{3} M_n$$

$$\begin{bmatrix} \frac{4}{5} & -\frac{2}{3} \\ -\frac{4}{5} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} D_n \\ M_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This has a trivial solution if $D_n = 0$, $M_n = 0$, but we know that $D_n + M_n = 1$. So we are looking for a non-trivial solution. Note also that the coefficient matrix is singular.

$$0 = \frac{4}{5} \times D_n - \frac{2}{3} \times M_n$$

$$D_n + M_n = 1$$

gives,

$$D_n = \frac{5}{11}$$

Extra notes for the student:

If one was going to find what the market share would be in 2004

$$\begin{bmatrix} D_n \\ M_n \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{3} \\ \frac{4}{5} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{6} \\ \frac{5}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{53}{90} \\ \frac{37}{90} \end{bmatrix}$$

One would use this number to find the market share in 2005 and so on. Eventually the market share would stabilize. But that would be a lenthier way to solve the problem.

6. Three kids - Jim, Corey and David receive an inheritance of $2,253,453. The money is put in three trusts but is not divided equally to begin with. Corey's trust is three times that of David's because Corey made an A in Dr. Kaw's class. Each trust is put in an interest generating investment. The three trusts of Jim, Corey and David pays an interest of 6%, 8%, 11%, respectively. The total interest of all the three trusts combined at the end of the first year is $190,740.57. The equations to find the trust money of Jim (*J*), Corey (*C*) and David (*D*) in a matrix form is

(A) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0.06 & 0.08 & 0.11 \end{bmatrix}\begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 190{,}740.57 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0.06 & 0.08 & 0.11 \end{bmatrix}\begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 190{,}740.57 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 6 & 8 & 11 \end{bmatrix}\begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 190{,}740.57 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 6 & 8 & 11 \end{bmatrix}\begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 19{,}074{,}057 \end{bmatrix}$

Solution

The correct answer is (B).

Let *J*, *C*, and *D* be the inheritance portions of Jim, Corey and David, respectively.

The total inheritance is $2,253,453 gives

$$J + C + D = \$2{,}253{,}453$$

Corey's trust is three times that of David's

$$C = 3D$$

gives

$$C - 3D = 0$$

The three trusts of Jim, Corey and David pay an interest of 6%, 8%, 11%, respectively. The total interest of all the three trusts combined at the end of the first year is $190,740.57.

The total interest is

$$\frac{6}{100}J + \frac{8}{100}C + \frac{11}{100}D = \$190{,}740.57$$

gives

$$0.06J + 0.08C + 0.11D = \$190{,}740.57$$

Three equations can be made from the information given

$$J + C + D = \$2{,}253{,}453$$

$$C - 3D = 0$$

$$0.06J + 0.08C + 0.11D = \$190{,}740.57$$

Setting the three equations in matrix form is as follows

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0.06 & 0.08 & 0.11 \end{bmatrix} \begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 190{,}740.57 \end{bmatrix}$$