Problem Set

Chapter 04.07
LU Decomposition

1. Show that LU decomposition is computationally a more efficient way of finding the inverse of a square matrix than using Gaussian elimination.

2. Use LU decomposition to find [L] and [U]
$$4x_1 + x_2 - x_3 = -2$$
$$5x_1 + x_2 + 2x_3 = 4$$
$$6x_1 + x_2 + x_3 = 6$$

3. Find the inverse of
$$[A] = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix}$$
using LU decomposition.

4. Fill in the blanks for the unknowns in the LU decomposition of the matrix given below
$$\begin{bmatrix} 25 & 5 & 4 \\ 75 & 7 & 16 \\ 12.5 & 12 & 22 \end{bmatrix} = \begin{bmatrix} \ell_{11} & 0 & 0 \\ \ell_{21} & \ell_{22} & 0 \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix} \begin{bmatrix} 25 & 5 & 4 \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

5. Show that the nonsingular matrix
$$[A] = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$
cannot be decomposed into LU form.

6.   The LU decomposition of

$$[A] = \begin{bmatrix} 4 & 1 & -1 \\ 5 & 1 & 2 \\ 6 & 1 & 1 \end{bmatrix}$$

is given by

$$\begin{bmatrix} 4 & 1 & -1 \\ 5 & 1 & 2 \\ 6 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1.25 & 1 & 0 \\ 1.5 & 2 & 1 \end{bmatrix} \begin{bmatrix} ?? & ?? & ?? \\ 0 & ?? & ?? \\ 0 & 0 & ?? \end{bmatrix}$$

Find the upper triangular matrix in the above decomposition?