Problem Set

Chapter 04.10
Eigenvalues and Eigenvectors

1. The eigenvalues λ of matrix $[A]$ are found by solving the equation(s)?
   (A) $[A][X]=[I]$
   (B) $[A][X]-\lambda[I]=0$
   (C) $|A=0|$
   (D) $|A-\lambda I|=0$

2. Find the eigenvalues and eigenvectors of
$$[A]=\begin{bmatrix}10 & 9\\ 2 & 3\end{bmatrix}$$
using the determinant method

3. Find the eigenvalues and eigenvectors of
$$[A]=\begin{bmatrix}4 & 0 & 1\\ -2 & 0 & 1\\ 2 & 0 & 1\end{bmatrix}$$
using the determinant method

4. Find the eigenvalues of these matrices by inspection
   (A) $\begin{bmatrix}2 & 0 & 0\\ 0 & -3 & 0\\ 0 & 0 & 6\end{bmatrix}$
   (B) $\begin{bmatrix}3 & 5 & 7\\ 0 & -2 & 1\\ 0 & 0 & 0\end{bmatrix}$
   (C) $\begin{bmatrix}2 & 0 & 0\\ 3 & 5 & 0\\ 2 & 1 & 6\end{bmatrix}$

5. Find the largest eigenvalue in magnitude and its corresponding vector by using the power method

$$[A]=\begin{bmatrix} 4 & 0 & 1 \\ -2 & 0 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

Start with an initial guess of the eigenvector as

$$\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix}$$

6. Prove if λ is an eigenvalue of $[A]$, then $\frac{1}{\lambda}$ is an eigenvalue of $[A]^{-1}$.

7. Prove that square matrices $[A]$ and $[A]^{\mathrm{T}}$ have the same eigenvalues.

8. Show that $\left|\det(A)\right|$ is the product of the absolute values of the eigenvalues of $[A]$.

Answers to Selected Problems:

1. D
2. $(12,1), \begin{bmatrix} 0.9762 \\ 0.2169 \end{bmatrix}, \begin{bmatrix} 0.8381 \\ -0.8381 \end{bmatrix}$
3. $(0,4,5615,0.43845), \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.87193 \\ -0.27496 \\ 0.48963 \end{bmatrix}, \begin{bmatrix} -0.27816 \\ 3.5284 \\ 0.99068 \end{bmatrix}$
4.
   (A) $2, -3, 6$
   (B) $3, -2, 0$
   (C) $2, 5, 6$
5. $4.5615, \begin{bmatrix} 1 \\ -0.31534 \\ 0.56154 \end{bmatrix}$ after 4 iterations