Problem Set

Chapter 04.04
Unary Matrix Operations

1. Let
$$[A]=\begin{bmatrix}25 & 3 & 6\\ 7 & 9 & 2\end{bmatrix}.$$
Find $[A]^{\mathrm{T}}$

2. If $[A]$ and $[B]$ are two $n\times n$ symmetric matrices, show that $[A]+[B]$ is also symmetric. Hint: Let $[C]=[A]+[B]$

3. Give an example of a 4×4 symmetric matrix.

4. Give an example of a 4×4 skew-symmetric matrix.

5. What is the trace of
(A) $[A]=\begin{bmatrix}7 & 2 & 3 & 4\\ -5 & -5 & -5 & -5\\ 6 & 6 & 7 & 9\\ -5 & 2 & 3 & 10\end{bmatrix}$
(B) For
$$[A]=\begin{bmatrix}10 & -7 & 0\\ -3 & 2.099 & 6\\ 5 & -1 & 5\end{bmatrix}$$
Find the determinant of $[A]$ using the cofactor method.

6. $\det(3[A])$ of a $n\times n$ matrix is
(A) $3\det(A)$
(B) $3\det(A)$
(C) $3^n\det(A)$
(D) $9\det(A)$

7. For a 5×5 matrix $[A]$, the first row is interchanged with the fifth row, the determinant of the resulting matrix $[B]$ is
(A) $\det(A)$
(B) $-\det(A)$

(C) $5\det(A)$
(D) $2\det(A)$

8. $\det\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ is

(A) 0
(B) 1
(C) -1
(D) ∞

9. Without using the cofactor method of finding determinants, find the determinant of

$$[A] = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 3 & 5 \\ 6 & 9 & 2 \end{bmatrix}$$

10. Without using the cofactor method of finding determinants, find the determinant of

$$[A] = \begin{bmatrix} 0 & 0 & 2 & 3 \\ 0 & 2 & 3 & 5 \\ 6 & 7 & 2 & 3 \\ 6.6 & 7.7 & 2.2 & 3.3 \end{bmatrix}$$

11. Without using the cofactor method of finding determinants, find the determinant of

$$[A] = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 2 & 5 & 6 & 0 \\ 1 & 2 & 3 & 9 \end{bmatrix}$$

12. Given the matrix

$$[A] = \begin{bmatrix} 125 & 25 & 5 & 1 \\ 512 & 64 & 8 & 1 \\ 1157 & 89 & 13 & 1 \\ 8 & 4 & 2 & 1 \end{bmatrix}$$

and

$\det(A) = -32400$

find the determinant of

(A) $[A]=\begin{bmatrix} 125 & 25 & 5 & 1 \\ 512 & 64 & 8 & 1 \\ 1141 & 81 & 9 & -1 \\ 8 & 4 & 2 & 1 \end{bmatrix}$

(B) $[A]=\begin{bmatrix} 125 & 25 & 1 & 5 \\ 512 & 64 & 1 & 8 \\ 1157 & 89 & 1 & 13 \\ 8 & 4 & 1 & 2 \end{bmatrix}$

(C) $[B]=\begin{bmatrix} 125 & 25 & 5 & 1 \\ 1157 & 89 & 13 & 1 \\ 512 & 64 & 8 & 1 \\ 8 & 4 & 2 & 1 \end{bmatrix}$

(D) $[C]=\begin{bmatrix} 125 & 25 & 5 & 1 \\ 1157 & 89 & 13 & 1 \\ 8 & 4 & 2 & 1 \\ 512 & 64 & 8 & 1 \end{bmatrix}$

(E) $[D]=\begin{bmatrix} 125 & 25 & 5 & 1 \\ 512 & 64 & 8 & 1 \\ 1157 & 89 & 13 & 1 \\ 16 & 8 & 4 & 2 \end{bmatrix}$

13. What is the transpose of

$$[A]=\begin{bmatrix} 25 & 20 & 3 & 2 \\ 5 & 10 & 15 & 25 \\ 6 & 16 & 7 & 27 \end{bmatrix}$$

14. What values of the missing numbers will make this a skew-symmetric matrix?

$$[A]=\begin{bmatrix} 0 & 3 & ? \\ ? & 0 & ? \\ 21 & ? & 0 \end{bmatrix}$$

15. What values of the missing number will make this a symmetric matrix?

$$[A]=\begin{bmatrix} 2 & 3 & ? \\ ? & 6 & 7 \\ 21 & ? & 5 \end{bmatrix}$$

16. Find the determinant of

$$[A]=\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 5 \end{bmatrix}$$

17. What is the determinant of an upper triangular matrix $[A]$ that is of order $n \times n$?

18. Given the determinant of

$$[A]=\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & a \end{bmatrix}$$

is -564, find a.

19. Why is the determinant of the following matrix zero?

$$[A]=\begin{bmatrix} 0 & 0 & 0 \\ 2 & 3 & 5 \\ 6 & 9 & 2 \end{bmatrix}$$

20. Why is the determinant of the following matrix zero?

$$[A]=\begin{bmatrix} 0 & 0 & 2 & 3 \\ 0 & 2 & 3 & 5 \\ 6 & 7 & 2 & 3 \\ 6.6 & 7.7 & 2.2 & 3.3 \end{bmatrix}$$

21. Show that if $[A][B]=[I]$, where $[A]$, $[B]$ and $[I]$ are matrices of $n \times n$ size and $[I]$ is an identity matrix, then $\det(A) \neq 0$ and $\det(B) \neq 0$.

Answers to Selected Problems

1. $\begin{bmatrix} 25 & 7 \\ 3 & 9 \\ 6 & 2 \end{bmatrix}$

2. $c_{ij} = a_{ij} + b_{ij}$ for all *i, j*.
   and
   $c_{ji} = a_{ji} + b_{ji}$ for all *i, j*.
   $c_{ji} = a_{ij} + b_{ij}$ as $[A]$ and $[B]$ are symmetric
   Hence $c_{ji} = c_{ij}$.
3.
4.

5. a) 19
   b) -150.05

6. C

7. A

8. C

9. 0: Can you answer why?

10. 0: Can you answer why?

11. $5 \times 3 \times 6 \times 9 = 810$: Can you answer why?
12. –32400 b) 32400 c) 32400 d) –32400 e) –64800
13. $[A]^T = \begin{bmatrix} 25 & 5 & 6 \\ 20 & 10 & 16 \\ 3 & 15 & 7 \\ 2 & 25 & 27 \end{bmatrix}$

14. $\begin{bmatrix} 0 & 3 & -21 \\ -3 & 0 & 4 \\ 21 & -4 & 0 \end{bmatrix}$

15. $\begin{bmatrix} 2 & 3 & 21 \\ 3 & 6 & 7 \\ 21 & 7 & 5 \end{bmatrix}$

16. The determinant of $[A]$ is

$$25\begin{bmatrix} 8 & 1 \\ 12 & 5 \end{bmatrix} - 5\begin{bmatrix} 64 & 1 \\ 144 & 5 \end{bmatrix} + 1\begin{bmatrix} 64 & 8 \\ 144 & 12 \end{bmatrix}$$
$$= 25(28) - 5(176) + 1(-384)$$
$$= -564$$

17. The determinant of an upper triangular matrix is the product of its diagonal elements, $\prod_{i=1}^{n} a_{ii}$

18. $\det(A) = -120a + 36$
$120a + 36 = 564$
$a = 5$

19. The first row of the matrix is zero, hence, the determinant of the matrix is zero.

20. Row 4 of the matrix is 1.1 times Row 3. Hence, its determinant is zero.

21. We know that $\det(AB) = \det(A)\det(B)$.
$[A][B] = [I]$
$\det(AB) = \det(I)$

$$\det(I) = \prod_{i=1}^{n} a_{ii} = \prod_{i=1}^{n} 1 = 1$$
$\det(A)\det(B) = 1$

Therefore,
$\det(A) \neq 0$ and
$\det(B) \neq 0$.