Problem Set

Chapter 04.03
Binary Matrix Operations

1. For the following matrices

$$[A]=\begin{bmatrix}3 & 0\\ -1 & 2\\ 1 & 1\end{bmatrix},\ [B]=\begin{bmatrix}4 & -1\\ 0 & 2\end{bmatrix},\ [C]=\begin{bmatrix}5 & 2\\ 3 & 5\\ 6 & 7\end{bmatrix}$$

Find where possible
   (A) $4[A]+5[C]$
   (B) $[A][B]$
   (C) $[A]=2[C]$

2. Food orders are taken from two engineering departments for a takeout. The order is tabulated below.
Food order:

$$\begin{matrix} & \scriptstyle\textit{Chicken Sandwich} & \scriptstyle\textit{Fries} & \scriptstyle\textit{Drink} \\ \textit{Mechanical} & 25 & 35 & 25 \\ \textit{Civil} & 21 & 20 & 21 \end{matrix}$$

However they have a choice of buying this food from three different restaurants. Their prices for the three food items are tabulated below

Price Matrix:

$$\begin{matrix} & \scriptstyle\textit{McFat} & \scriptstyle\textit{Burcholestrol} & \scriptstyle\textit{Kentucky Sodium} \\ \textit{Chicken Sandwich} & 2.42 & 2.38 & 2.46 \\ \textit{Fries} & 0.93 & 0.90 & 0.89 \\ \textit{Drink} & 0.95 & 1.03 & 1.13 \end{matrix}$$

Show how much each department will pay for their order at each restaurant. Which restaurant would be more economical to order from for each department?

3. Given

$$[A]=\begin{bmatrix}2 & 3 & 5\\ 6 & 7 & 9\\ 2 & 1 & 3\end{bmatrix}$$

$$[B]=\begin{bmatrix}3 & 5\\ 2 & 9\\ 1 & 6\end{bmatrix}$$

$$[C]=\begin{bmatrix}5 & 2\\ 3 & 9\\ 7 & 6\end{bmatrix}$$

Illustrate the distributive law of binary matrix operations

$$[A]([B]+[C])=[A][B]+[A][C]$$

4. Let $[I]$ be a $n\times n$ identity matrix. Show that $[A][[I]=[I][A]=[A]$ for every $n\times n$ matrix $[A]$.

   Let $[C]_{n\times n}=[A]_{n\times n}[I]_{n\times n}$

5. Consider there are only two computer companies in a country. The companies are named *Dude* and *Imac*. Each year, company *Dude* keeps 1/5th of its customers, while the rest switch to *Imac*. Each year, *Imac* keeps 1/3rd of its customers, while the rest switch to *Dude*. If in 2002, *Dude* has 1/6th of the market and *Imac* has 5/6th of the market.
   (A) What is the distribution of the customers between the two companies in 2003? Write the answer first as multiplication of two matrices.
   (B) What would be distribution when the market becomes stable?

6. Given

$$[A]=\begin{bmatrix}12.3 & -12.3 & 10.3\\ 11.3 & -10.3 & -11.3\\ 10.3 & -11.3 & -12.3\end{bmatrix},$$

$$[B]=\begin{bmatrix}2 & 4\\ -5 & 6\\ 11 & -20\end{bmatrix}$$

   $[A][B]$ matrix size is ________________

7. Given

$$[A]=\begin{bmatrix}12.3 & -12.3 & 10.3\\ 11.3 & -10.3 & -11.3\\ 10.3 & -11.3 & -12.3\end{bmatrix},$$

$$[B]=\begin{bmatrix}2 & 4\\ -5 & 6\\ 11 & -20\end{bmatrix}$$

   if $[C]=[A][B]$, then $c_{31}=$ ______________________

Answers to Selected Problems

1.

(A) $=\begin{bmatrix} 37 & 10 \\ 11 & 33 \\ 34 & 39 \end{bmatrix}$

(B) $=\begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}$

(C) $=\begin{bmatrix} -7 & -4 \\ -7 & -8 \\ -11 & -13 \end{bmatrix}$

2. The cost in dollars is 116.80, 116.75, 120.90 for the Mechanical Department at three fast food joints. So BurCholestrol is the cheapest for the Mechanical Department. The cost in dollars is 89.37, 89.61, 93.19 for the Civil Department at three fast food joints. McFat is the cheapest for the Civil Department.

3. $[B]+[C]=\begin{bmatrix} 3 & 5 \\ 2 & 9 \\ 1 & 6 \end{bmatrix}+\begin{bmatrix} 5 & 2 \\ 3 & 9 \\ 7 & 6 \end{bmatrix}$

$=\begin{bmatrix} 8 & 7 \\ 5 & 18 \\ 8 & 12 \end{bmatrix}$

$[A]([B]+[C])=\begin{bmatrix} 71 & 128 \\ 155 & 276 \\ 45 & 68 \end{bmatrix}$

$[A][B]=\begin{bmatrix} 17 & 67 \\ 41 & 147 \\ 11 & 37 \end{bmatrix}$

$[A][C]=\begin{bmatrix} 54 & 61 \\ 114 & 129 \\ 34 & 31 \end{bmatrix}$

$[A][B]+[A][C]=\begin{bmatrix} 71 & 128 \\ 155 & 276 \\ 45 & 68 \end{bmatrix}$

4. Hint: $c_{ij} = \sum_{p=1}^{n} a_{ip} i_{pj}$

$= a_{i1} i_{1j} + \ldots\ldots + a_{i,j-1} i_{j-1,j} + a_{ij} i_{jj} + a_{i(j+1)} i_{(j+1)j} + \ldots\ldots + a_{in} i_{nj}$

Since

$$i_{ij} = 0 \text{ for } i \neq j$$
$$= 1 \text{ for } i = j$$
$$c_{ij} = a_{ij}$$

So $[A] = [A][I]$

Similarly do the other case

$[I][A] = [A]$. Just do it!

5.

(A) At the end of 2002, Dude has

$$\frac{1}{5} \times \frac{1}{6} + \frac{2}{3} \times \frac{5}{6} = 0.589 .$$

Imac has

$$\frac{4}{5} \times \frac{1}{6} + \frac{1}{3} \times \frac{5}{6} = 0.411$$

In matrix form $\begin{bmatrix} \frac{1}{5} & \frac{2}{3} \\ \frac{4}{5} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{6} \\ \frac{5}{6} \end{bmatrix} = \begin{bmatrix} 0.589 \\ 0.411 \end{bmatrix}$

(B) Stable distribution is [10/22 12/22] (Try to do this part of the problem first by finding the distribution five years from now).

6. 3×2

$(10.3 \times 2) + ((-5) \times (-11.3)) + (11 \times (-12.3)) = -58.2$