Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Chapter 4.04
Unary Matrix Operations
COMPLETE SOLUTION SET

1. If the determinant of a 4×4 matrix $[A]$ is given as 20, then the determinant of $5[A]$ is
    (A) 100
    (B) 12500
    (C) 25
    (D) 62500

Solution
The correct answer is (B).

The determinant of $k[A]$ where $[A]$ is an $n \times n$ matrix and k is a constant, is given by

$$\det(k[A]) = k^n \det(A)$$

$$\begin{aligned}\det(5[A]) &= 5^4 \det(A)\\ &= 625 \times (20)\\ &= 12500\end{aligned}$$

2. If the matrix product $[A][B][C]$ is defined, then $([A][B][C])^T$ is

(A) $[C]^T [B]^T [A]^T$

(B) $[A]^T [B]^T [C]^T$

(C) $[A][B][C]^T$

(D) $[A]^T [B][C]$

Solution

The correct answer is (*A*).

If $[A][D]$ is a matrix product, then

$$([A][D])^T = [D]^T [A]^T$$

Now if $[D] = [B][C]$, then

$$\begin{aligned}([A][B][C])^T &= ([B][C])^T [A]^T \\ &= [C]^T [B]^T [A]^T\end{aligned}$$

3. The trace of a matrix

$$\begin{bmatrix} 5 & 6 & -7 \\ 9 & -11 & 13 \\ -17 & 19 & 23 \end{bmatrix}$$

is

(A) 17
(B) 39
(C) 40
(D) 110

Solution

The correct answer is (A).

The trace of then matrix is given by

$$tr(A) = \sum_{i=1}^{3} a_{ii}$$

$$= \sum_{i=1}^{3} a_{ii}$$

$$= a_{11} + a_{22} + a_{33}$$

$$= 5 + (-11) + 23$$

$$= 17$$

4. A square $n \times n$ matrix $[A]$ is symmetric if

(A) $a_{ij} = a_{ji}, i = j$ for all i, j

(B) $a_{ij} = a_{ji}, i \neq j$ for all i, j

(C) $a_{ij} = -a_{ji}, i = j$ for all i, j

(D) $a_{ij} = -a_{ji}, i \neq j$ for all i, j

Solution

The correct answer is (B).

A square matrix $[A]$ is symmetric if $a_{ij} = a_{ji}$ for all i, j. This is also the same as saying if $[A]^T = [A]$, then $[A]$ is a symmetric matrix. An example of a 4×4 symmetric matrix is

$$\begin{bmatrix} 6 & 7 & -5 & 13 \\ 7 & 17 & 19 & 71 \\ -5 & 19 & 23 & 73 \\ 13 & 71 & 73 & 31 \end{bmatrix}$$

5. The determinant of the matrix

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & 3 & 8 \\ 0 & 9 & a \end{bmatrix}$$

is 50. The value of a is then

(A) 0.6667
(B) 24.67
(C) -23.33
(D) 5.556

Solution

The correct answer is (*B*).

The determinant of the 3×3 matrix $[A]$ can be found by using the co-factor method. We use the first column of the matrix for the cofactor method because of several zeros to make the computation easier.

$$\begin{aligned} \det\begin{bmatrix} 25 & 5 & 1 \\ 0 & 3 & 8 \\ 0 & 9 & a \end{bmatrix} &= 25\begin{vmatrix} 3 & 8 \\ 9 & a \end{vmatrix} - 0\begin{vmatrix} 5 & 1 \\ 9 & a \end{vmatrix} + 0\begin{vmatrix} 5 & 1 \\ 3 & 8 \end{vmatrix} \\ &= 25(3 \times a - 8 \times 9) + 0 + 0 \\ &= 25(3a - 72) \end{aligned}$$

Since

$$\begin{aligned} \det(A) &= 50 \\ 25(3a - 72) &= 50 \\ 3a - 72 &= \frac{50}{25} \\ &= 2 \end{aligned}$$

$$\begin{aligned} a &= \frac{72 + 2}{3} \\ &= 24.67 \end{aligned}$$

6. $[A]$ is a 5×5 matrix and a matrix $[B]$ is obtained by the row operations of replacing $Row1$ with $Row3$, and then $Row3$ is replaced by a linear combination of $2 \times Row3 + 4 \times Row2$. If $\det(A) = 17$, then $\det(B)$ is equal to
   (A) 12
   (B) -34
   (C) -112
   (D) 112

Solution
The correct answer is (*B*).

By exchanging $Row1$ with $Row3$, the determinant changes sign. $Row3$ is multiplied by 2, so the determinant of the resulting matrix will double. The addition of a multiple of a different row to $Row3$ does not change the determinant.

Hence

$$\det(B) = (17)(-1)(2) = -34$$