Multiple-Choice Test

Chapter 04.07
LU Decomposition Method

1.   The $[L][U]$ decomposition method is computationally more efficient than Naïve Gauss elimination for solving
    (A)   a single set of simultaneous linear equations.
    (B)   multiple sets of simultaneous linear equations with different coefficient matrices and the same right hand side vectors.
    (C)   multiple sets of simultaneous linear equations with the same coefficient matrix and different right hand side vectors.
    (D)   less than ten simultaneous linear equations.

2.   The lower triangular matrix $[L]$ in the $[L][U]$ decomposition of the matrix given below

$$\begin{bmatrix} 25 & 5 & 4 \\ 10 & 8 & 16 \\ 8 & 12 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.7333 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 6 & 14.400 \\ 0 & 0 & -4.2400 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 10 & 1 & 0 \\ 8 & 12 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.5000 & 1 \end{bmatrix}$

3.   The upper triangular matrix $[U]$ in the $[L][U]$ decomposition of the matrix given below

$$\begin{bmatrix} 25 & 5 & 4 \\ 0 & 8 & 16 \\ 0 & 12 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.7333 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 6 & 14.400 \\ 0 & 0 & -4.2400 \end{bmatrix}$

(C) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 8 & 16 \\ 0 & 0 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0.2000 & 0.16000 \\ 0 & 1 & 2.4000 \\ 0 & 0 & -4.240 \end{bmatrix}$

4.   For a given 2000×2000 matrix $[A]$, assume that it takes about 15 seconds to find the inverse of $[A]$ by the use of the $[L][U]$ decomposition method, that is, finding the $[L][U]$ once, and then doing forward substitution and back substitution 2000 times using the 2000 columns of the identity matrix as the right hand side vector. The approximate time, in seconds, that it will take to find the inverse if found by repeated use of the Naive Gauss elimination method, that is, doing forward elimination and back substitution 2000 times by using the 2000 columns of the identity matrix as the right hand side vector is most nearly
(A) 300
(B) 1500
(C) 7500
(D) 30000

5. The algorithm for solving a set of n equations $[A][X] = [C]$, where $[A] = [L][U]$ involves solving $[L][Z] = [C]$ by forward substitution. The algorithm to solve $[L][Z] = [C]$ is given by

(A) $z_1 = c_1 / l_{11}$
    for i from 2 to n do
    sum $= 0$
    for j from 1 to i do
    sum $=$ sum $+ l_{ij} * z_j$
    end do
    $z_i = (c_i - sum) / l_{ii}$
    end do

(B) $z_1 = c_1 / l_{11}$
    for i from 2 to n do
    sum $= 0$
    for j from 1 to $(i-1)$ do
    sum $=$ sum $+ l_{ij} * z_j$
    end do
    $z_i = (c_i - sum) / l_{ii}$
    end do

(C) $z_1 = c_1 / l_{11}$
    for i from 2 to n do
    for j from 1 to $(i-1)$ do
    sum $=$ sum $+ l_{ij} * z_j$
    end do
 $z_i = (c_i - sum) / l_{ii}$
    end do

(D) for i from 2 to n do
    sum $= 0$
    for j from 1 to $(i-1)$ do
    sum $=$ sum $+ l_{ij} * z_j$
    end do
    $z_i = (c_i - sum) / l_{ii}$
    end do

6.   To solve boundary value problems, a numerical method based on finite difference method is used. This results in simultaneous linear equations with tridiagonal coefficient matrices. These are solved using a specialized $[L][U]$ decomposition method.

Choose the set of equations that approximately solves the boundary value problem

$$\frac{d^2y}{dx^2} = 6x - 0.5x^2,\ y(0)=0,\ y(12)=0,\ 0 \le x \le 12$$

The second derivative in the above equation is approximated by the second order accurate central divided difference approximation as learned in the differentiation module (Chapter 02.02). A step size of $h = 4$ is used, and hence the value of y can be found approximately at equidistantly placed 4 nodes between $x=0$ and $x=12$.

$$
\begin{array}{cccc}
i = 1 & i = 2 & i = 3 & i = 4 \\
\bullet & \bullet & \bullet & \bullet \\
x = 0 & x = 4 & x = 8 & x = 12
\end{array}
$$
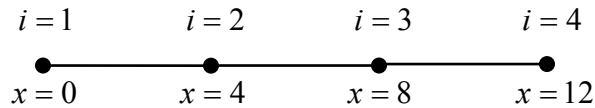
(A)
$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0.0625 & 0.125 & 0.0625 & 0 \\
0 & 0.0625 & 0.125 & 0.0625 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 16.0 \\ 16.0 \\ 0 \end{bmatrix}
$$

(B)
$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0.0625 & -0.125 & 0.0625 & 0 \\
0 & 0.0625 & -0.125 & 0.0625 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 16.0 \\ 16.0 \\ 0 \end{bmatrix}
$$

(C)
$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0.0625 & -0.125 & 0.0625 & 0 \\
0 & 0.0625 & -0.125 & 0.0625
\end{bmatrix}
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 16.0 \\ 16.0 \end{bmatrix}
$$

(D)
$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0.0625 & 0.125 & 0.0625 & 0 \\
0 & 0.0625 & 0.125 & 0.0625
\end{bmatrix}
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 16.0 \\ 16.0 \end{bmatrix}
$$

For a complete solution, refer to the links at the end of the book.