Problem Set

Chapter 04.02
Vectors

1. For

$$\vec{A} = \begin{bmatrix} 2 \\ 9 \\ -7 \end{bmatrix}, \vec{B} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}, \vec{C} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

   find $\vec{A} + \vec{B}$ and $2\vec{A} - 3\vec{B} + \vec{C}$.

2. Are

$$\vec{A} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \vec{B} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \vec{C} = \begin{bmatrix} 1 \\ 4 \\ 25 \end{bmatrix}$$

   linearly independent?.
   What is the rank of the above set of vectors?

3. Are

$$\vec{A} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \vec{B} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \vec{C} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$$

   linearly independent?.
   What is the rank of the above set of vectors?

4. Are

$$\vec{A} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \vec{B} = \begin{bmatrix} 2 \\ 4 \\ 10 \end{bmatrix}, \vec{C} = \begin{bmatrix} 1.1 \\ 2.2 \\ 5.5 \end{bmatrix}$$

   linearly independent?
   What is the rank of the above set of vectors?

5. If a set of vectors contains the null vector, the set of vectors is linearly
   (A) Independent
   (B) Dependent?

6. If a set of vectors is linearly independent, a subset of the vectors is linearly
   (A) Independent.
   (B) Dependent.

7. If a set of vectors is linearly dependent, then
   (A) At least one vector can be written as a linear combination of others.
   (B) At least one vector is a null vector.

8. If the dimension of a set of vectors is less than the number of vectors in the set, then the set of vectors is linearly
   (A) Dependent.
   (B) Independent.

9. Find the dot product of $\vec{A} = (2,1,2.5,3)$ and $\vec{B} = (-3,2,1,2.5)$

10. If $\vec{u}, \vec{v}, \vec{w}$ are three nonzero vector of 2-dimensions, then
   (A) $\vec{u}, \vec{v}, \vec{w}$ are linearly independent
   (B) $\vec{u}, \vec{v}, \vec{w}$ are linearly dependent
   (C) $\vec{u}, \vec{v}, \vec{w}$ are unit vectors
   (D) $k_1\vec{u} + k_2\vec{v} + k_3\vec{v} = \vec{0}$ has a unique solution.

11. $\vec{u}$ and $\vec{v}$ are two non-zero vectors of dimension n. Prove that if $\vec{u}$ and $\vec{v}$ are linearly dependent, there is a scalar q such that $\vec{v} = q\vec{u}$.

12. $\vec{u}$ and $\vec{v}$ are two non-zero vectors of dimension n. Prove that if there is a scalar q such that $\vec{v} = q\vec{u}$, then $\vec{u}$ and $\vec{v}$ are linearly dependent.

Answers to Selected Problems:

1. $\begin{bmatrix} 5 \\ 11 \\ -2 \end{bmatrix}$; $\begin{bmatrix} -4 \\ 13 \\ -28 \end{bmatrix}$

2. 3

3. 3

4. No;1

5. B

6. A

7. A

8. A

9. 6

10. B

11. Hint :
    Start with $k_1\vec{u} + k_2\vec{v} = \vec{0}$
    Show that $k_1 \neq 0$ and $k_2 \neq 0$ because $\vec{u}$ and $\vec{v}$ are both nonzero.
    Hence

$$\vec{v} = -\frac{k_1}{k_2}\vec{u}$$

$$= q\vec{u}\ . \qquad \left(q = -\frac{k_1}{k_2}\right)$$

12. Hint:
    Since

$$\vec{v} = q\vec{u}$$

$$\vec{v} - q\vec{u} = \vec{0}$$

$q \neq 0$, otherwise $\vec{v} = \vec{0}$

So the equation

$$k_1\vec{v} + k_2\vec{u} = \vec{0}$$

has a non trivial solution of
$k_1 = 1, k_2 = q \neq 0$.